UNIT 6

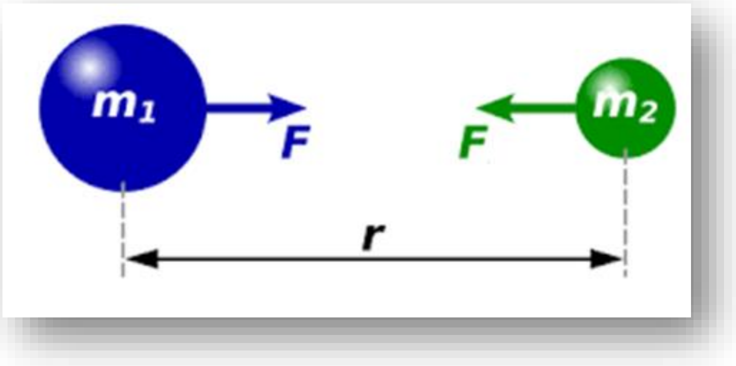
GRAVITATION

NEWTON'S LAW OF GRAVITATION

Everybody in the universe attracts every other body with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centres

EXPLANATION

Consider two bodies A and B of masses m_1 and m_2 respectively that are placed with their centers at distance r from each other as shown in figure



According to the statement force of attraction between two bodies is directly proportional to the product of their masses. Therefore,

$$F \propto m_1 m_2 \quad ....(i)$$

The gravitational force of attraction is inversely proportional to the square of the distance between the
centers of the masses of the bodies. Therefore

$$F \propto \frac{1}{r^2} \quad ....(ii)$$

Combining equation (i) and (ii)

$$F \propto \frac{m_1 m_2}{r^2}$$

$$F = G \frac{m_1 m_2}{r^2}$$

The proportionality constant "G" is called the universal gravitational constant

THE GRAVITATIONAL CONSTANT "G":

The proportionality constant "G" is called the universal gravitational constant. Lord Cavendish first determined its value, and the presently accepted value is

$$G = 6.673 \times 10^{-11}\ N\text{-}m^2 / Kg^2$$

DIFFERENCE BETWEEN “G” AND “g”

The difference between G and g is listed in the below table.

Parameter	G (Universal Gravitational Constant)	g (Acceleration Due to Gravity)
Definition	G is a fundamental constant in physics. It appears in Newton's law of universal gravitation.	g is a measure of how quickly objects accelerate toward the Earth's surface due to the force of gravity.
Symbol	G	g
Value	Approximately 6.674×10^{-11} N m^2 / kg^2	Approximately 9.81 (m/s^2) near the surface of the Earth.
Dependence	G is a universal constant and does not change with the location or objects involved.	g varies with location and is stronger and closer to the Earth's center.
Role	Determines the strength of the gravitational force between two objects with mass.	Governs the acceleration experienced by objects in a gravitational field, such as near the Earth's surface.

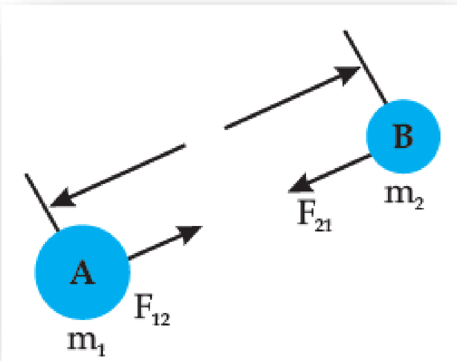
KEY POINTS

The gravitational force has the following characteristics:

i) It is always present between every two objects because of their masses.
ii) It exists everywhere in the universe.
iii) It forms an action-reaction pair.
iv) It is independent of the medium between the objects.
v) It is directly proportional to the product of the masses of objects.
vi) It is inversely proportional to the square of the distance between the centres of the objects.
vii) Hence it follows the “Inverse Square Law”.

LAW OF GRAVITATION AND NEWTON’S THIRD LAW OF MOTION

According to Newton's law of gravitation, every two objects attract each other with equal force but in opposite direction. As shown in fig



From the figure:

$m_1 \rightarrow Mass\ of\ body\ A$
$m_2 \rightarrow Mass\ of\ body\ B$
$F_{12} \rightarrow froce\ with\ which\ body\ A\ attracts\ body\ B$
$F_{21} \rightarrow froce\ with\ which\ body\ B\ attracts\ body\ A$

Then according to this law

$$\mathbf{F_{12} = -\,F_{21}}$$

This shows that, the two forces are equal in magnitude but opposite in direction. Now, if F_{12} is considered as “Action Force” and F_{21} as “Reaction Force”. Then by using above equation, it is concluded that “Action equals to reaction but in opposite direction”. Recall that, above statement is in accordance with the Newton's third law of motion which states that

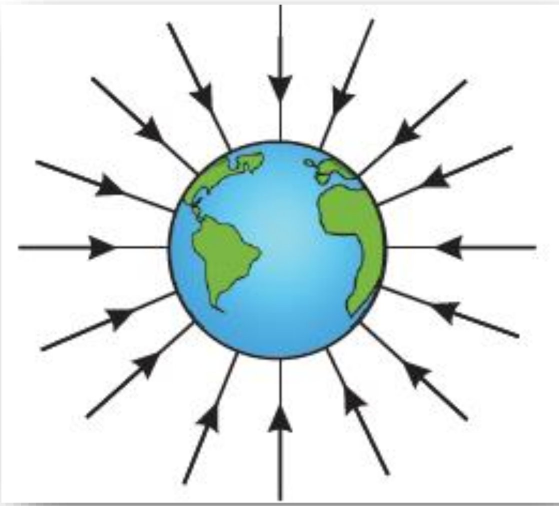
“To every action there is always an equal and opposite reaction”.

Hence, Newton's law of gravitation is consistent with Newton's third law of motion.

GRAVITATIONAL FIELD

“A gravitational field is a region in which a mass experiences a force due to gravitational attraction”.

The earth has an attractive gravitational field around it. Any object near the Earth experience this force which is due to Earth's gravity. This field is directed towards the centre of the Earth as shown in fig



GRAVITATIONAL FIELD STRENGTH

Gravitational field strength ‘g’ is the gravitational force acting per unit mass.

The gravitational field strength “g” is approximately 10 Newton per kilogram $10Nkg^{-1}$.

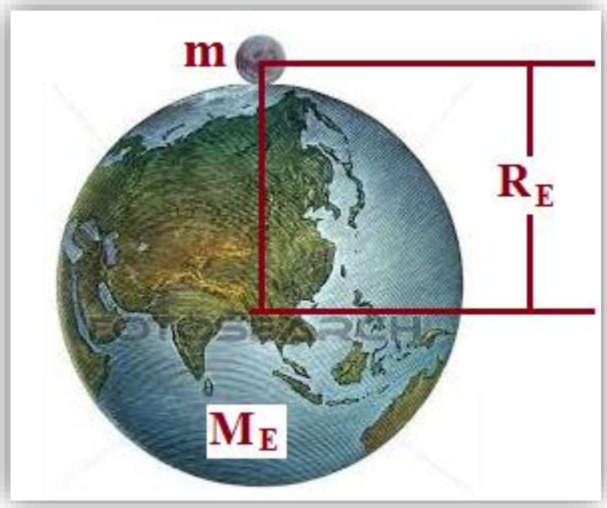
WEIGHT

The weight of an object is the measurement of gravitational force acting on the object.

Weight 'W' of an object of mass 'm', in a gravitational field of strength 'g' is given by the relation:

$$W = mg$$

MASS OF EARTH



Let,

m = mass of the ball is placed on the surface of earth

M_e = mass of the earth

R_E = radius of the earth

Then according to Newton's law of universal gravitation, the gravitational force F of the Earth acts on the ball is:

$$F = G\frac{m\ Me}{R_e^2} \dots\dots\dots (i)$$

Whereas the force with which Earth attracts the ball towards its centre is equal to the weight of the ball. Therefore

$$F = W(\text{weight})$$
$$F = m\,g \dots\dots (ii)$$

Comparing equation (i) and (ii)

$$mg = G\frac{mM_E}{R_e^2}$$

$$g = \frac{GM_E}{R_e^2}$$

$$M_e = \frac{g}{G}R_e^2$$

By putting the value of

$g = 9.8\ m/s^2$

$G = 6.67 \times 10^{-11}\ N.m^2/Kg^2$

$R_E = 6.38 \times 10^6$ m

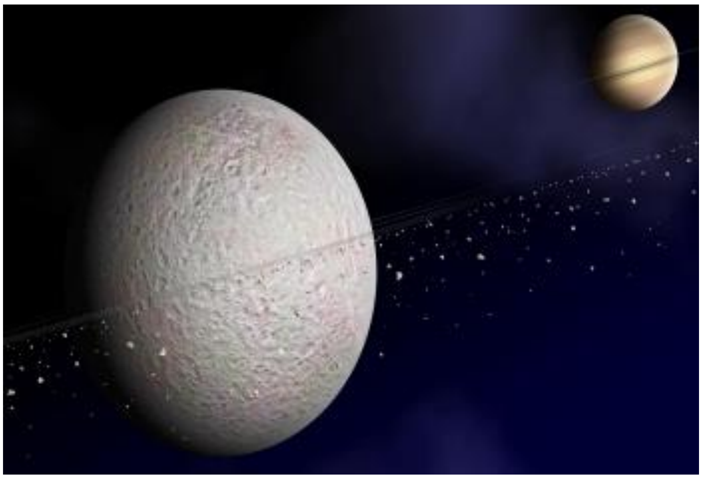
$$M_E = \frac{(9.8)(6.38x10^6)^2}{(6.67x10^{-11})}$$

$M_E = 5.98 \times 10^{24}$ kg

ARTIFICIAL SATELLITES

A satellite is an object that revolves around a planet. Satellites are of two types:

1. Natural satellites (Fig-1)
2. Artificial satellites (Fig-2)

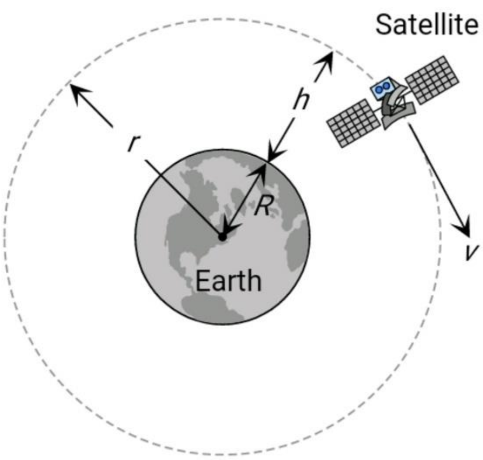
NATURAL SATELLITES	ARTIFICIAL SATELLITES
The planet revolves around another planet naturally is called “Natural Satellite”.	The objects that are sent into space by scientists to revolve around the Earth or other planets are called Artificial Satellite”.
Example The moon is a natural satellite because it revolves around the Earth naturally	Example Sputnik-1, Explorer-1 are amongst the artificial satellites

Artificial satellites are used for different purposes like

- For communication.
- For making star maps.
- For making maps of planetary surfaces.
- For collecting weather information.
- For taking pictures of planets, etc.

NEWTON’S LAW OF GRAVITATION IN THE MOTION OF SATELLITE

artificial satellite revolves around a planet is called an “orbit. Rockets are used to put satellites into orbits in space.



Let us consider the motion of a satellite revolving around the Earth; as shown in the figure:

The gravitational pull of Earth on the satellite provides the centripetal force needed to keep a satellite in orbit around some planet

$$\text{Centripetal force} = \text{Gravitational force}$$

$$F_C = F_G \dots\dots\dots (i)$$

Centripetal force is given as

$$F_C = \frac{m\,v^2}{r}$$

and gravitation force is

$$F_G = \frac{G\,m\,M}{r^2}$$

Substituting the values of F and F in equation (i):

$$\frac{m\,v^2}{r} = \frac{G\,m\,M}{r^2}$$

$$\frac{m\,v^2}{r} = \frac{G\,m\,M}{r \times r}$$

$$v^2 = \frac{G\;\;M}{r} \qquad \{r = R + h\}$$

$$v^2 = \frac{G\;\;M}{R + h}$$

$$v = \sqrt{\frac{G\,M}{R + h}}$$

This gives the velocity that a satellite must possess when orbiting around Earth in an orbit of radius (r = R + h). This shows that the speed of the satellite is independent of its mass.

TIME REQUIRED OF A SATELLITE

The time required for a satellite to complete one revolution around the Earth in its orbit is called its time period "T". The time period of a satellite can be calculated as:

$$T = \frac{2\,\pi\,r}{v}$$

The velocity of the satellite is given by

$$v = \sqrt{\frac{G\,M}{R + h}}$$

$$v = \sqrt{\frac{G\,M}{r}}$$

Substituting the expression of velocity in equation (i)

$$T = \frac{2\,\pi\,r}{\sqrt{\frac{G\,M}{r}}}$$

$$T = 2\,\pi\;\frac{\sqrt{r^2}}{\sqrt{\frac{G\,M}{r}}}$$

$$T = 2\pi\sqrt{\frac{r^2}{\frac{GM}{r}}}$$

$$T = 2\pi\sqrt{r^2 \times \frac{r}{GM}}$$

$$T = 2\pi\sqrt{\frac{r^3}{GM}}$$

ORBITAL VELOCITY

The velocity required to keep the satellite into its orbit is called "Orbital Velocity"

Consider a satellite with mass m is moving around the earth in an orbit of radius r and the velocity of the satellite is a tangent to the path of the orbit, and the net centripetal force acting on the satellite is .

$$F_c = \frac{m\ v^2}{r} \ldots\ldots\ldots\ldots..(i)$$

This net centripetal force is equal to the weight of the satellite

$$W_S = mg_h \quad \ldots\ldots(ii)$$

Now

$$F_C = W_S \quad \ldots\ldots.(iii)$$

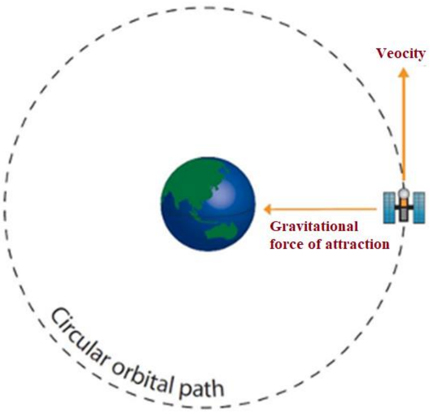
Substituting the values of 'F_C' and 'W_S' in equation (i):

$$\frac{m\ v^2}{r} = mg_h$$

$$v^2 = \mathrm{r}\, g_h$$

$$v = \sqrt{\mathrm{r}\, g_h} \quad \{r = R + h\}$$

$$v = \sqrt{(\mathrm{R}+\mathrm{h})\, g_h}$$



If satellite is orbiting very close to the surface of Earth then: h << R

In this case orbital radius may be considered equal to radius of Earth.

Therefore, $R + h = R$

Also $g = g_h$

and $V = V_C$

Where,

V ⟶ Critical velocity **C**

g ⟶ Acceleration due to gravity on the surface of Earth.

In terms of above factors above equation becomes:

$$v_c = \sqrt{g\,R}$$

This is known as "Critical velocity". It is also known as orbital speed or proper speed.

CRITICAL VELOCITY

The constant horizontal velocity is required to put the satellite into a stable circular orbit around the Earth.

CRITICAL SPEED OF THE SATELLITE GET CLOSER TO THE EARTH

$$v_c = \sqrt{g\,R}$$

$$v_c = \sqrt{(10)\,(6.38 \times 10^6)}$$

$$v_c = 7.99 \times 10^3\, m/s$$

$$v_c = 7.99\; k\,m/s$$

It should be noted that as the satellite get closer to the Earth, the gravitational pull of the Earth on it gets stronger.